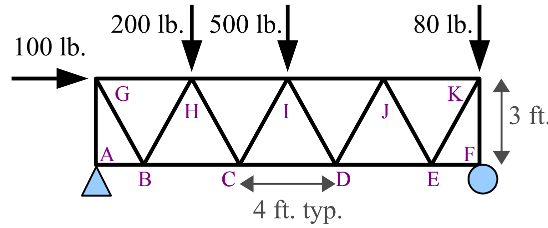
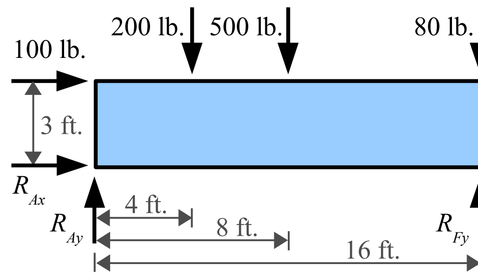
Method of Joints



The Method of Joints is used to calculate the tensile or compressive forces in all the members of a truss.

Problem Find the forces in members of this truss.



Step 1 Calculate reaction forces at the supports, using three equations: $\Sigma M=0$, $\Sigma F_x=0$, and $\Sigma F_y=0$. Treat the truss as a solid body, since reaction forces depend only on external forces and dimensions.

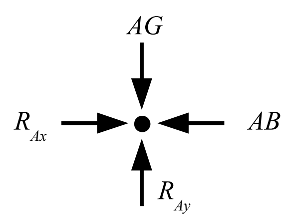
$$\Sigma M_A=0=-3\,\text{ft.}(100\,\text{lb.})-4\,\text{ft.}(200\,\text{lb.})-8\,\text{ft.}(500\,\text{lb.})$$
$$-16\,\text{ft.}(80\,\text{lb.})+16\,\text{ft.}\,R_{Fy}$$

$$R_{Fy}=\frac{3\,\text{ft.}(100\,\text{lb.})+4\,\text{ft.}(200\,\text{lb.})+8\,\text{ft.}(500\,\text{lb.})+16\,\text{ft.}(80\,\text{lb.})}{16\,\text{ft.}}$$
$$=398.75\,\text{lb.}$$

$$\Sigma F_y=0=R_{Ay}+R_{Fy}-200\,\text{lb.}-500\,\text{lb.}-80\,\text{lb.}$$
$$R_{Ay}=-398.75\,\text{lb.}+200\,\text{lb.}+500\,\text{lb.}+80\,\text{lb.}=381.25\,\text{lb.}$$
$$\Sigma F_x=0=100\,\text{lb.}+R_{Ax}\rightarrow R_{Ax}=-100\,\text{lb.}$$

Since R_{Ax} is negative, the arrow is drawn backwards. Draw the arrow the same way in all subsequent diagrams, and use $R_{Ax}=-100\,\text{lb.}$

Joint **A**



Step 2 Draw all of the forces acting on a single joint. Select a joint with known applied forces or reaction forces. At joint **A**, you may not know in advance whether forces *AG* and *AB* are in tension (pulling on the joint) or in compression (pushing on the joint). Make a guess, and the sign of the result (+/-) will show if the guess was right.

Step 3 Use $\Sigma F_x=0$ and $\Sigma F_y=0$ to solve for the unknown forces. In the equation, all forces acting upward or to the right are positive; forces acting downward or to the left are negative.

$$\Sigma F_x=0=R_{Ax}-AB \quad\rightarrow\quad AB=R_{Ax}=100\,\text{lb.}$$

AB is positive, so the arrow is drawn correctly.

AB = 100 lb. tension.

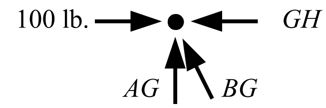
$$\Sigma F_y=0=R_{Ay}+-AG \quad\rightarrow\quad AG=R_{Ay}=381.25\,\text{lb.}$$

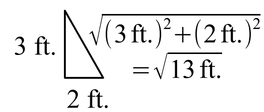
AG is positive, so the arrow is drawn correctly.

AG = 381.25 lb. compression.

Step 4 Select an adjacent joint, and repeat Steps 2 & 3. Continue until all forces are known.

Joint **G**





Force AG is drawn at Joint **A** as a compressive force, so it must also be drawn at Joint **G** as a compressive force.

The vertical component of BG is $\frac{3}{\sqrt{13}}BG$.

The horizontal component of BG is $\frac{2}{\sqrt{13}}BG$.

$$\Sigma F_y = 0 = AG - \frac{3}{\sqrt{13}}BG \;\rightarrow\; BG = \frac{AG}{3/\sqrt{13}} = \frac{381.25\,\text{lb.}}{3/\sqrt{13}} = 458.2\,\text{lb.}$$

The answer is positive, so BG = 458.2 lb. tension.

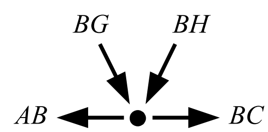
$$\Sigma F_x = 0 = -GH + 100\,\text{lb.} + \frac{2}{\sqrt{13}}BG$$

$$GH = 100\,\text{lb.} + \frac{2}{\sqrt{13}}BG = 100\,\text{lb.} + \frac{2}{\sqrt{13}}458.2\,\text{lb.} = 354.2\,\text{lb.}$$

The answer is positive, so GH = 354.2 lb. compression.

Now consider the adjoining joint **B**, because we already know two of the forces acting on it.

Joint **B**



$$\Sigma F_y = 0 = \frac{-3}{\sqrt{13}}BG - \frac{3}{\sqrt{13}}BH \;\rightarrow\; BH = -BG = -458.2\,\text{lb.}$$

The answer is negative, so BH = 458.2 lb. tension.

$$\Sigma F_x = 0 = -AB + \frac{2}{\sqrt{13}}BG - \frac{2}{\sqrt{13}}BH + BC$$

$$BC = AB - \frac{2}{\sqrt{13}}BG + \frac{2}{\sqrt{13}}BH$$

$$= 100\,\text{lb.} - \frac{2}{\sqrt{13}}458.2\,\text{lb.} + \frac{2}{\sqrt{13}}(-458.2\,\text{lb.}) = 100\,\text{lb.}$$

The answer is positive, so BC = 100 lb. tension.

Signs

The Method of Joints uses positive and negative signs for two different purposes.

[1] When assigning signs to forces in the force balance equations, positive is up & right, negative is down & left.

[2] The result of the force balance equation is positive when the arrow is drawn correctly; negative when the arrow is drawn backwards.

Symmetry

If the the truss is symmetrical, and the loading is symmetrical, then the forces in each half will also be symmetrical. You need only solve for joints **A**, **B**, **C**, **G**, **H**, and **I** to find all forces, because AG=FK, AB=EF, BH=EJ, and so forth.